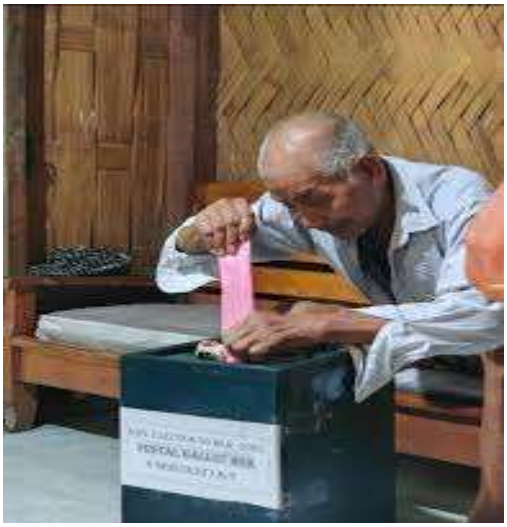
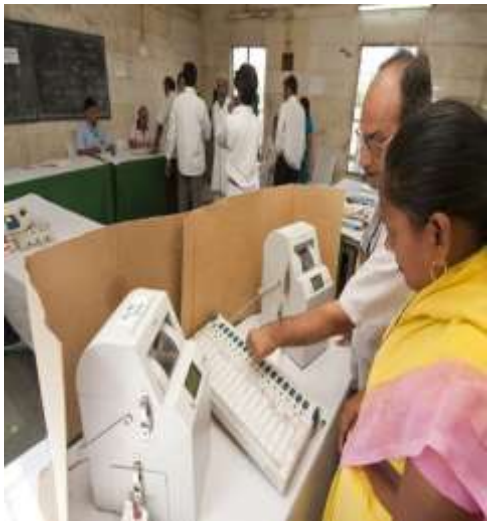
THE VILLAGE INTERNATIONAL SCHOOL
"We Nurture Dreams"

CLASS12 Max.Marks: 20

12-06-24 MATHEMATICS(041) – MT1 Time: 50 minutes

General Instructions:
1) Questions 1 to 4 carries 1 mark each.
2) Questions 5 to 8 carries 2 marks each.
3) Questions 9 and 10 carries 4 marks each.

	SECTION A	
1.	Where is f(x) = [x] , -1 < x < 2 is not differentiable? **a)** -1 & 2 b) (-1,2) c) 0 & 1 d) 1 & 2	1
2.	A is a matrix of order 3x3 , such that $\|A\|$ = -4. Find $\|A \cdot adjA\|$ a) 16 b) -64 c) -16 d) 64	1
3.	$A = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 0 & 1 \\ 4 & 5 & 6 \end{bmatrix}$ if A_{ij} is the cofactor of a_{ij}, find $a_{11}A_{21}+a_{12}A_{22}+a_{13}A_{23}$ a)43 b) -43 c) -87 d) 0	1
4.	**If** $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ **, find** $\frac{dy}{dx}$ **a)** $\frac{2}{1+x^2}$ b) $\frac{2}{1-x^2}$ c) $\frac{2}{\sqrt{1-x^2}}$ d) $\frac{-2}{\sqrt{1+x^2}}$	1
	SECTION B	
5.	$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2-5A+7I = O$. Hence find A^{-1}.	2
6.	$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix}$ If $(A+B)^2 = A^2+B^2$, find a and b.	2
7.	$If\ x^y = y^x$, find $\frac{dy}{dx}$	2
8.	If f(x) = $\begin{cases} 5, x \le 2 \\ ax+b, 2 < x < 10 \\ 21, x \ge 10 \end{cases}$ is continuous , find the values of a and b	2

	SECTION C	
9.	x = a(cost+tsint) , y = a(sint – tcost) . Find $\frac{d^2y}{dx^2}$.	4
10.	In a legislative assembly election, a political party hired a public relations firm to promote its candidate in 3 ways : telephone , house calls and letters.The cost per contact (in paise) is given in Matrix A as   Cost per contact A = $\begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix}$ The no. of contact of each type made in 2 cities X and Y is given by Telephone Housecall Letter B = $\begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10{,}000 \end{bmatrix} \begin{matrix} \rightarrow X \\ \rightarrow Y \end{matrix}$ i) Find the total amount spent by the group in the city X ii) Find the total amount spent by the group in the city Y iii) Find AB or BA if possible **(OR)** Two schools P and Q want to award their selected students on the values of Tolerance, Kindness and leadership. The school P wants to award Rs x each , Rs y each and Rs z each for the three respective values to 3 , 2 and 1 students respectively with a total award money Rs 2200. School Q wants to spend Rs 3100 to award its 4,1 and 3 students on the respective values (by giving the same award money to the three values as school P) . If the total amount of award for one prize on each value is Rs 1200, using matrices &, find the answer for the following.	1 1 2

	 (i) What is the award money for Tolerance? (ii) What is the award money for Kindness? (iii) What is the award money for Leadership? (iv) Write the adjoint matrix of the 3x3 matrix used in this case.	1 1 1 1

ANSWERS

5)

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 5(3) & 5(1) \\ 5(-1) & 5(2) \end{bmatrix} + \begin{bmatrix} 7(1) & 7(0) \\ 7(0) & 7(1) \end{bmatrix} \qquad A^{-1}(A^2 - 5A + 7I) = A^{-1}O$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= O$$

$$A^{-1} = \frac{1}{7}(5I - A) \qquad \begin{bmatrix} 2/7 & -1/7 \\ 1/7 & 3/7 \end{bmatrix}$$

6) AB+ BA = O

AB = -BA

$$\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} a & -1 \\ b & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

$$-1\begin{bmatrix} a-b & 0 \\ 2a-b & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

a-b = -1

2a-b = 0

a = 1 b = 2

7)

$$\log(y^x) = \log(x^y)$$

$$x \cdot \log y = y \cdot \log x$$

$$\log y + \frac{x}{y} \cdot \frac{dy}{dx} = \frac{dy}{dx} \cdot \log x + \frac{y}{x}$$

$$\frac{x}{y} \cdot \frac{dy}{dx} - \frac{dy}{dx} \cdot \log x = \frac{y}{x} - \log y$$

$$\boldsymbol{\frac{dy}{dx} = \frac{y(y - x\log y)}{x(x - y\log x)}}$$

8) 2a+b = 5

10a+b = 21

8a = 16

a = 2 , b = 1

9)

$$\frac{dy}{dt} = a\left(\cos t - (\cos t + (-\sin t) \cdot t)\right)$$

$$\frac{dy}{dt} = a\left(\cos t - (\cos t - (\sin t) \cdot t)\right)$$

$$\frac{dy}{dt} = a\,(\cos t - \cos t + t \cdot \sin t)$$

$$\frac{dy}{dt} = a\,(0 + t\sin t)$$

$$\boldsymbol{\frac{dy}{dt} = a \cdot t \cdot \sin t}$$

$$\frac{dx}{dt} = a\left(-\sin t + (\sin t + \cos t \cdot t)\right)$$

$$\frac{dx}{dt} = a\,(-\sin t + \sin t + t \cdot \cos t)$$

$$\boldsymbol{\frac{dx}{dt} = a \cdot t \cdot \cos t}$$

dy/dx = tan t

$$\frac{d^2y}{dx^2} = \sec^2 t \div \boldsymbol{\frac{dx}{dt}}$$

$$\frac{d^2y}{dx^2} = \sec^2 t \div \boldsymbol{a \cdot t \cdot \cos t} \qquad \boldsymbol{\frac{d^2y}{dx^2} = \frac{\sec^3 t}{a \cdot t}}$$

10) i) RS300

ii) Rs 400

iii) Rs500

iv)